Cryptographic Hashes

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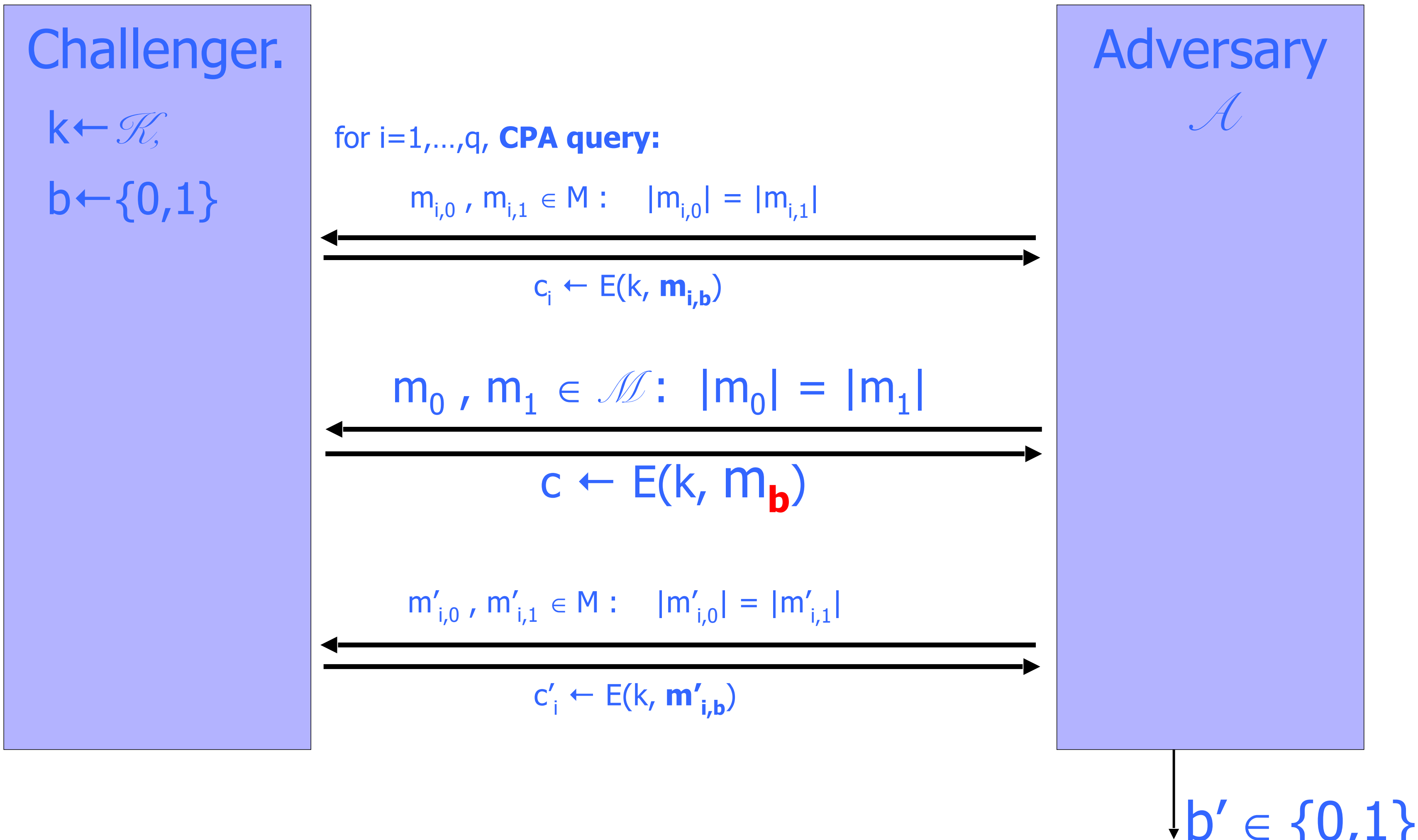
Recap: CPA

1. $k \leftarrow KeyGen(1^n)$. $b \leftarrow \{0,1\}$. Give Enc(k, ·) to $\mathscr{A}$.
2. $\mathscr{A}$ chooses as many plaintexts as he wants, and receives the corresponding ciphertexts via Enc(k, ·).
3. $\mathscr{A}$ picks two plaintexts M_0 and M_1. (Picking plaintexts for which A previously learned ciphertexts is allowed!)
4. $\mathscr{A}$ receives the ciphertext of M_b, and continues to have accesses to Enc(k, ·).
5. $\mathscr{A}$ outputs b'.

$\mathscr{A}$ wins if b'=b.

Recap: CPA



For all efficient adversary $\mathscr{A}$,

$|$ Pr[b=b'] - 1/2 $|$ is "negligible".

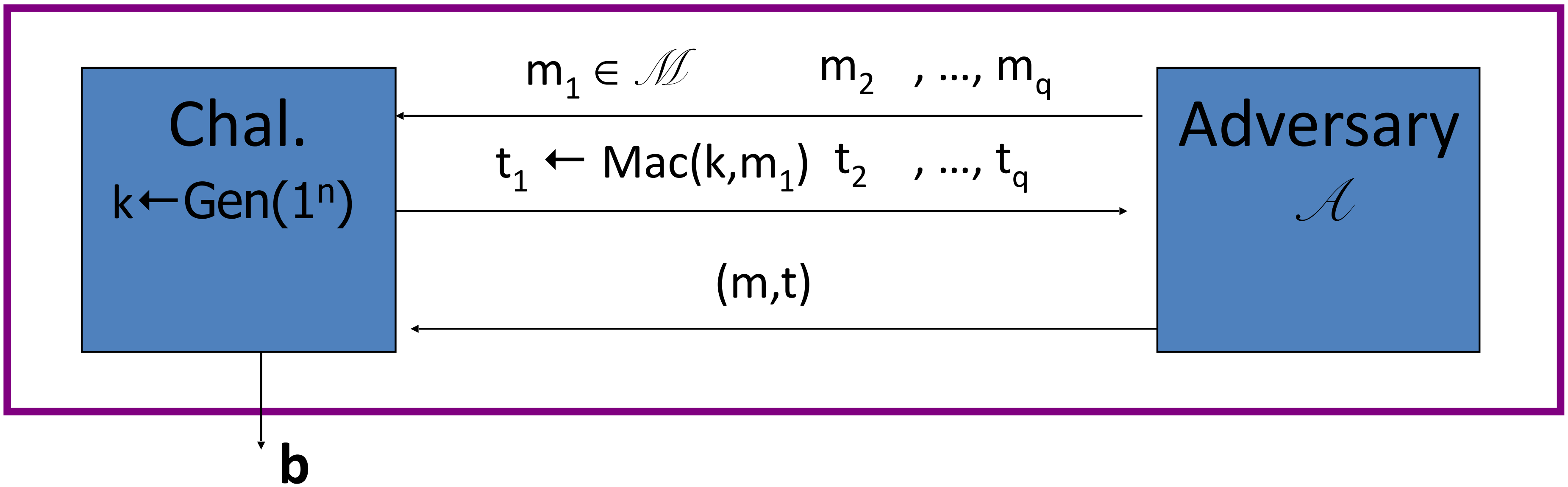
Recap: Message Integrity Game

1. $\texttt{k} \leftarrow \texttt{Gen}(1^n)$.
2. $\mathscr{A}$ is given polynomial time and an oracle access to query $\texttt{Mac}(\texttt{k},\cdot)$. Let $\texttt{t}_i = \texttt{Mac}(\texttt{k}, \texttt{m}_i)$ and $\texttt{Q} = \{(\texttt{m}_1, \texttt{t}_1), \ldots, (\texttt{m}_q, \texttt{t}_q)\}$.
3. $\mathscr{A}$ outputs (m, t).

$\mathscr{A}$ wins the game if $\texttt{Vrfy}(\texttt{m}, \texttt{t}) = 1$ and $(\texttt{m},\texttt{t}) \notin \texttt{Q}$.

Recap: Message Integrity

(Gen, Mac, Vrfy) --- a message authentication code scheme.

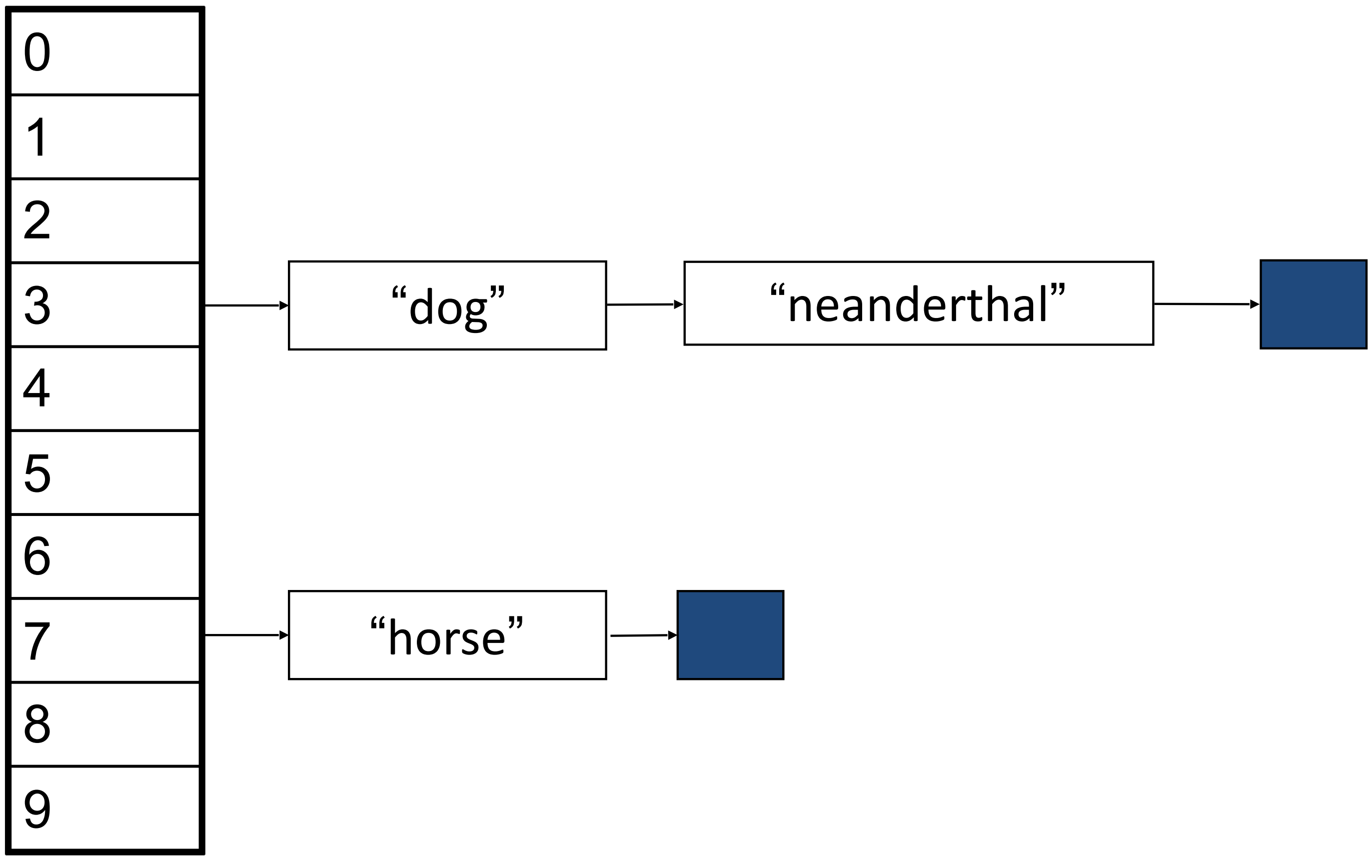


$$\mathbf{b} = \begin{cases} 1 & \text{if } Vrfy(k,m,t) = 1 \text{ and } (m,t) \notin \{ (m_1, t_1), \dots, (m_q, t_q) \} \\ 0 & \text{otherwise} \end{cases}$$

Def: (Gen, Mac, Vrfy) is a <u>**Secure Message Authentication Code**</u> if for all "efficient" $\mathcal{A}$:

$$Adv_{Mac}[\mathcal{A}] = \Pr[\text{Chal. outputs } 1] \text{ is "negligible."}$$

Normal CS Hashing



$$H\ (\text{char } s[\,]) = (s[0] - \text{'a'}) \bmod 10$$

Magic Function f

- One Way:
  - For every integer x, *easy* to compute $f(x)$
  - Given $f(x)$, *hard* to find any information about x
- Collision Resistant:
  - "Impossible" to find pair (x, y) where $x \neq y$ and $f(x) = f(y)$

Regular Hash Functions

1. Many-to-one: maps a large number of values to a small number of hash values
2. Evenly distributed: for typical data sets, $\Pr(H(x) = n) = 1/N$ where N is the number of hash values and n = 0 .. $N - 1$.
3. Efficient: $H(x)$ is easy to compute.

How well does

$$H\,(\text{char } s[\,]) = (s[0] - \text{‘a’}) \bmod 10$$

satisfy these properties?

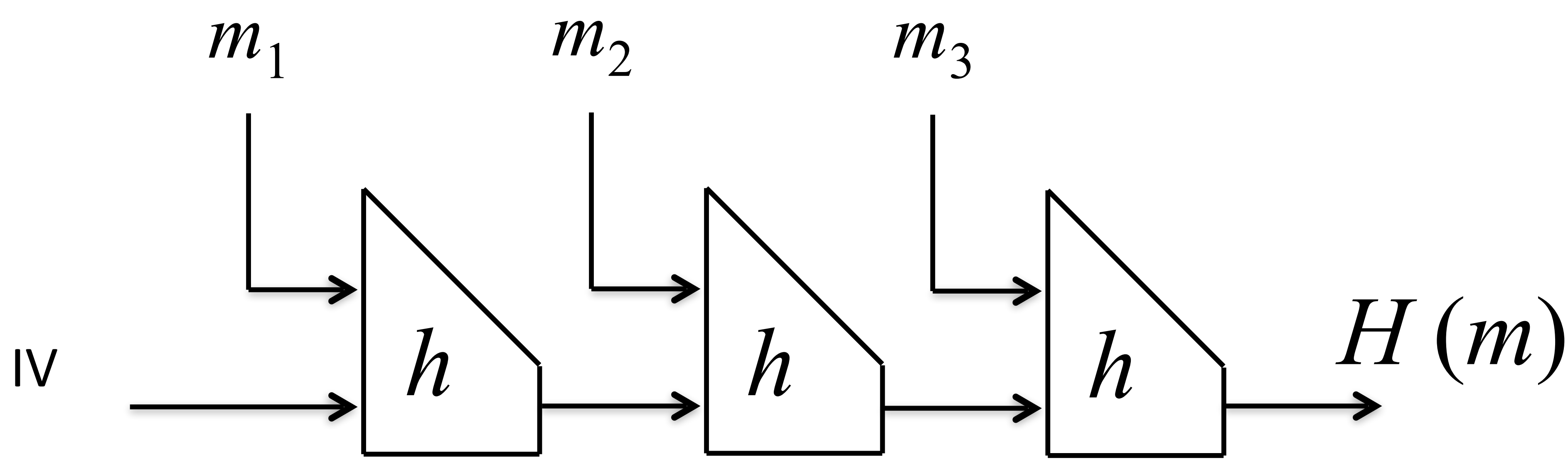
Cryptographic Hash Functions

Collision resistance (even for malicious adversary):

- **Preimage** resistance: for a uniformly chosen v, it is hard to find x such that $H(x) = v$.
- **Second-preimage** resistance: given x, it is hard to find $y \neq x$ such that $H(y) = H(x)$.
- **Collision resistance**: it is hard to find any x and y such that $y \neq x$ and $H(x) = H(y)$.

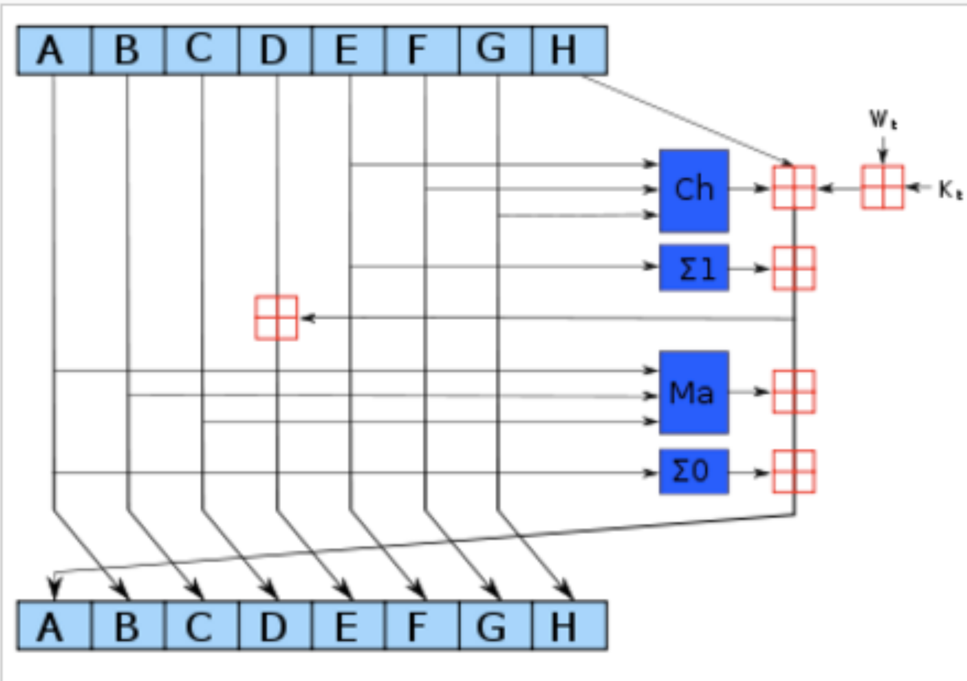
Merkle-Damgård Transform

$$m = (m_1, m_2, m_3)$$



Compressing by a single bit is as easy (or as hard) as compressing by an arbitrary amount.

Example Real World Hash Functions



One iteration in a SHA-2 family compression function. The blue components perform the following operations:

$$\mathrm{Ch}(E,F,G) = (E \wedge F) \oplus (\neg E \wedge G)$$

$$\mathrm{Ma}(A,B,C) = (A \wedge B) \oplus (A \wedge C) \oplus (B \wedge C)$$
$$\Sigma_0(A) = (A \ggg 2) \oplus (A \ggg 13) \oplus (A \ggg 22)$$
$$\Sigma_1(E) = (E \ggg 6) \oplus (E \ggg 11) \oplus (E \ggg 25)$$

The bitwise rotation uses different constants for SHA-512. The given numbers are for SHA-256. The red ⊞ is addition modulo 2^{32}.

- MD5 is broken
- SHA-1 is phasing out
- SHA-256
  - M-D Transform
  - 256-bit output
  - 512-bit block size
  - 64 rounds
  - a combination of AND, OR, XOR, ADD, RotR, ShR
  - 128 bit security

Authentication through Hash and Mac

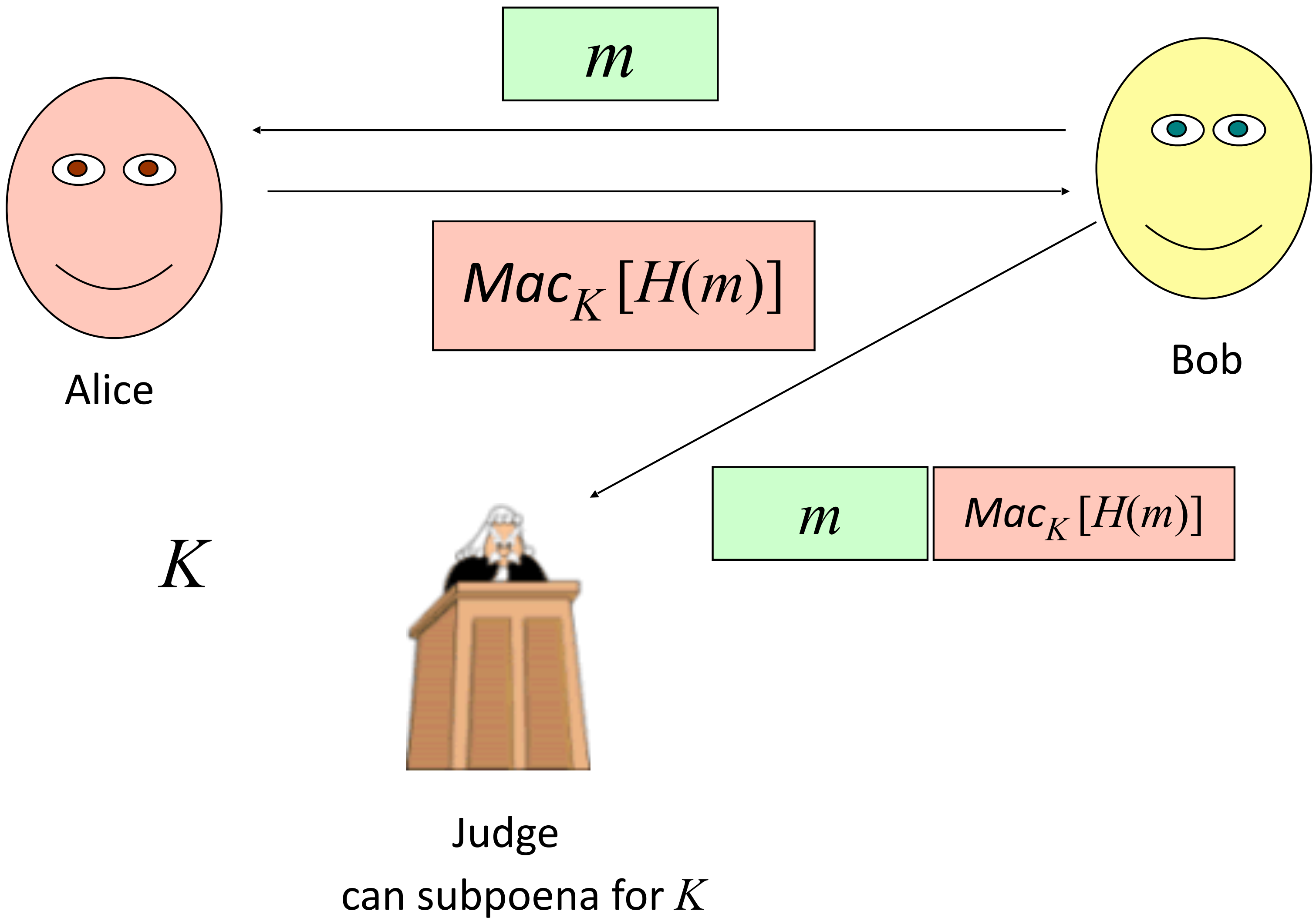
If (Gen’, Mac’, Vrfy’) is a MAC for fixed length messages,

- Gen: Gen’
- Mac: $t =$ Mac’$(k, H(m))$
- Vrfy: outputs 1 if and only if Vrfy’$(k, H(m), t) = 1$

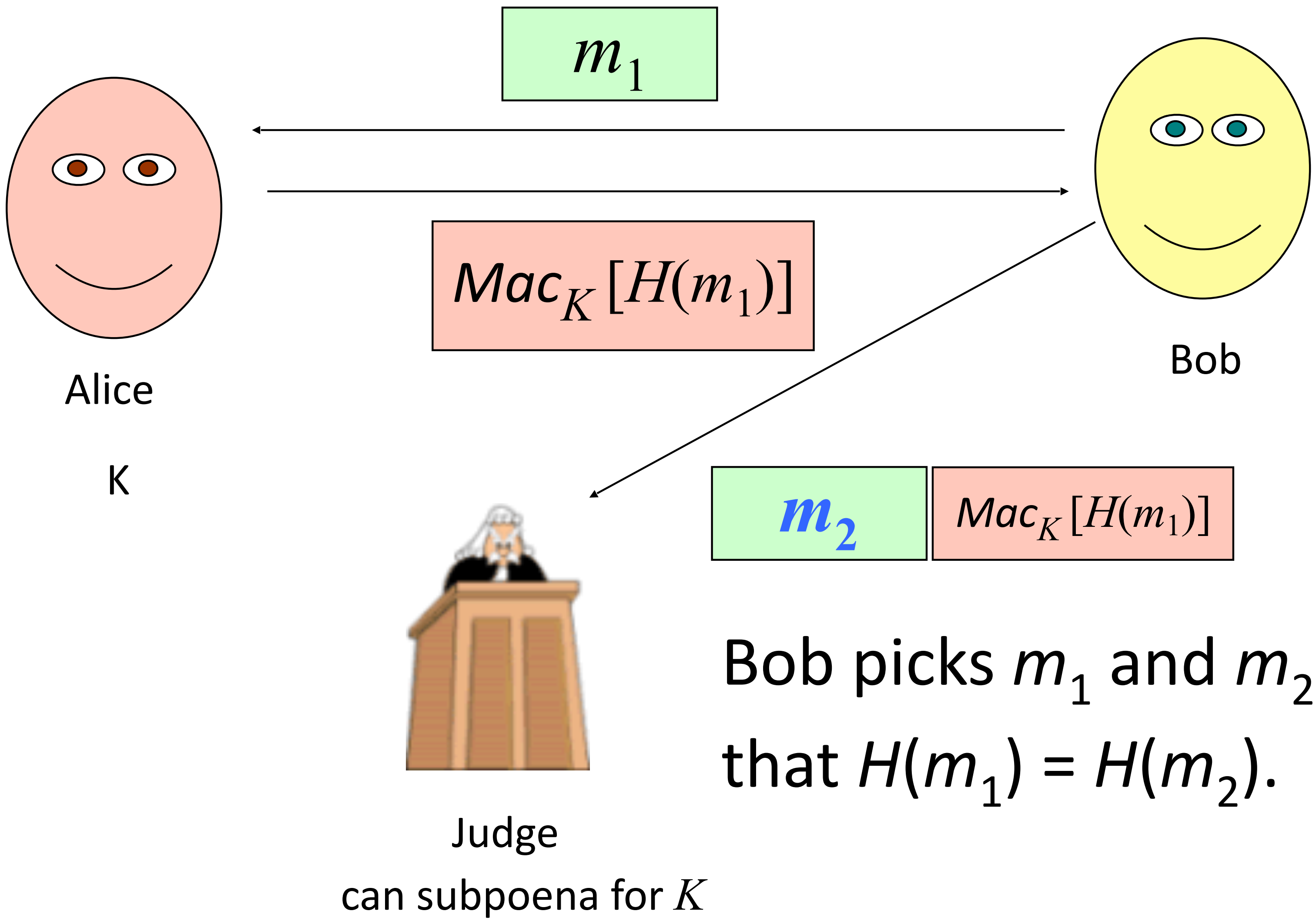
Other Applications of Hashing

- Fingerprinting
- Authenticated Data Structures
- Coin tossing

IOU Request Protocol



Attacking IOU Request Protocol



Bob picks m_1 and m_2 such that $H(m_1) = H(m_2)$.

Finding m_1 and m_2

Bob generates different agreeable m_1 messages:

```
I, {Alice | Alice Hacker | Alice P.
Hacker | Ms. A. Hacker},{owe | agree
to pay} Bob{the sum of | the amount
of}{$2 | $2.00 | 2 dollars | two
dollars}{by | before}{January 1st | 1
Jan | 1/1 | 1-1}{2016 | 2016 AD}.
```

How many different-text messages are there?

Finding m_1 and m_2

Bob generates 2^{10} different agreeable m_2 messages:

`I,{Alice | Alice Hacker | Alice P. Hacker | Ms. A. Hacker},{owe | agree to pay}Bob{the sum of | the amount of}{`**`$2 quadrillion | $2000000000000000 | 2 quadrillion dollars | two quadrillion dollars`**`}{by | before}{January 1`st` | 1 Jan | 1/1 | 1-1}{2016 | 2016 AD}.`

Bob's Quadrillionaire Plan

- For each message $m_{1,i}$ and $m_{2,i}$, Bob computes $H(m_{1,i})$ and $H(m_{2,i})$.
- If $H(m_{1,i}) = H(m_{2,j})$ for some i and j, Bob sends Alice $m_{1,i}$, gets $\mathrm{Mac}_K[H(m_{1,i})]$ back.
- Bob sends the judge $m_{2,j}$ and $\mathrm{Mac}_K[H(m_{1,i})]$.

Chances of Success

- Assume the Hash function H is good (uniform randomly distributed outcome)

What is the probability that $H(m_{1,i}) = H(m_{2,j})$ for some i and j ?

Birthday “Paradox”

What is the probability that two people in this room have the same birthday?

Birthday Paradox

Ways to assign k different birthdays without duplicates:

N = 365 * 364 * ... * (365 – k + 1)

= 365! / (365 – k)!

Ways to assign k different birthdays with possible duplicates:

D = 365 * 365 * ... * 365 = 365^k

Birthday “Paradox”

Assuming real birthdays assigned randomly:

N/D = probability there are no duplicates

1 - N/D = probability there is a duplicate

$$= 1 - 365! / ((365 - k)!(365)^k)$$

Generalizing Birthdays

$$\mathrm{P}(n, k) = 1 - \frac{n!}{(n-k)!\, n^k}$$

Given k random selections from n possible values, $\mathrm{P}(n, k)$ gives the probability that there is at least 1 duplicate.

Applying to Birthdays

- For n = 365, k = 20:

  P(365, 20) ≈ .4114

- For n = 365, k = 40:

  P (365, 40) ≈ .8912

Is 128 bits enough for hash output?

- For $n = 2^{128}$, $k = 2^{40}$: P $(2^{128}, 2^{40}) > 1.77 \times 10^{-15}$
- For $n = 2^{128}$, $k = 2^{60}$: P $(2^{128}, 2^{60}) > 1.95 \times 10^{-3}$
- For $n = 2^{128}$, $k = 2^{65}$: P $(2^{128}, 2^{60}) > 0.86$

A 10 thousand core machine can brute-force 2^{65} hashes in about 50 days (assuming 10^9 hashes per second on each core).

Assumes you hash function is perfect (e.g., MD5 was not broken merely as a result of bruteforce).

A Most Disturbing Program!

From https://freedom-to-tinker.com/blog/felten/report-crypto-2004/

```
#!/usr/bin/perl -w
use strict;
use Digest::MD5 qw(md5_hex);

# Create a stream of bytes from hex.
my @bytes1 = map {chr(hex($_))} qw(d1 31 dd 02 c5 e6 ee c4 69 3d 9a 06 98 af f9 5c 2f ca b5 87
12 46 7e ab 40 04 58 3e b8 fb 7f 89 55 ad 34 06 09 f4 b3 02 83 e4 88 83 25 71 41 5a 08 51 25 e8 f7
cd c9 9f d9 1d bd f2 80 37 3c 5b d8 82 3e 31 56 34 8f 5b ae 6d ac d4 36 c9 19 c6 dd 53 e2 b4 87 da
03 fd 02 39 63 06 d2 48 cd a0 e9 9f 33 42 0f 57 7e e8 ce 54 b6 70 80 a8 0d 1e c6 98 21 bc b6 a8 83
93 96 f9 65 2b 6f f7 2a 70);

my @bytes2 = map {chr(hex($_))} qw(d1 31 dd 02 c5 e6 ee c4 69 3d 9a 06 98 af f9 5c 2f ca b5 07
12 46 7e ab 40 04 58 3e b8 fb 7f 89 55 ad 34 06 09 f4 b3 02 83 e4 88 83 25 f1 41 5a 08 51 25 e8 f7
cd c9 9f d9 1d bd 72 80 37 3c 5b d8 82 3e 31 56 34 8f 5b ae 6d ac d4 36 c9 19 c6 dd 53 e2 34 87
da 03 fd 02 39 63 06 d2 48 cd a0 e9 9f 33 42 0f 57 7e e8 ce 54 b6 70 80 28 0d 1e c6 98 21 bc b6 a8
83 93 96 f9 65 ab 6f f7 2a 70);

# Print MD5 hashes
print md5_hex(@bytes1), "\n", md5_hex(@bytes2), "\n";
```

79054025255fb1a26e4bc422aef54eb4
79054025255fb1a26e4bc422aef54eb4